

## Uncertainty quantification for marginal computations

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The marginal likelihood (also called the evidence) plays a central role in Bayesian model selection. It is defined as the integral over the parameter space of the likelihood times the prior density. In this talk, we show that this integration problem can be rewritten as a classification task and how we can take benefit of that point. We consider the situation where the calculation of the likelihood is tractable.

We recall the noise-contrastive estimation method introduced by [1] and completed by [2]. The calculating of the integrated likelihood is reduced to the estimation of the intercept of a well-chosen pseudo-logistic regression model. We highlight that this strategy is equivalent the bridge sampling methodology [3] and [4] and we introduce a modified version of the weighted likelihood bootstrap approach [5] to derive measures of uncertainty on the estimation of the marginal likelihood. The weighted likelihood bootstrap approach is a well known method to approximate a posterior distribution for a parameter of interest defined via either an estimating equation or a likelihood, known to be robust to model misspecification. Our proposal is a modification of the weighted likelihood bootstrap algorithm associated to the pseudo-logistic regression model.

We show through some examples how effective our approach is in validating Bridge Sampling estimates of the marginal likelihood.

## References

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